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THE USE OF COMPUTATIONAL DIAGRAMS AND NOMOGRAMS IN HIGHER EDUCATION

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Institutional researchers are well aware of the effectiveness of graphical displays and, where circumstances allow, often use them in place of tabular data displays. There are many such instances where the graphical display is an efficient and preferable means of communication. However, the same characteristics that make the graph a superior presentation device also ensure its usefulness and acceptability as a computational tool. Field engineers and scientists in many disciplines use graphs to make quick, repetitive, and accurate calculations, a usage which has not been replaced by the hand calculator or microcomputer.

Computational Diagrams

The three-dimensional diagram. The common practice of relating variables to geometrical dimensions is probably the reason that graphs are not used more often for computation in higher education. Once four or more variables must be linked, our limited ability to deal with a world beyond the three dimensional brings graphing to an end. This may be illustrated by a simple example.

Suppose we consider an institution's tuition rate to be a decision variable (i.e., not fixed) and consider tuition revenue to be a function of *two* independent variables—enrollment and tuition rate. The standard graphical representation is three dimensional (Figure 1), a structure which taxes both the person constructing the graph and the user.

As illustrated, we enter the graph with an enrollment of 20,000 and a tuition rate of $1,500 and find the intersection on the tuition-enrollment plane. Moving vertically from this point to the intersection within the tuition-enrollment-revenue surface and from there horizontally to the revenue axis, we arrive at a revenue of $30M. Using the three-dimensional graph in this manner is more difficult than using the more common two-dimensional graph, but because of the linear relationships, an "answer" may be found via parallel lines. Once the relationship becomes non-linear, the surface develops a curvature and finding a reliable intersection is no longer feasible.

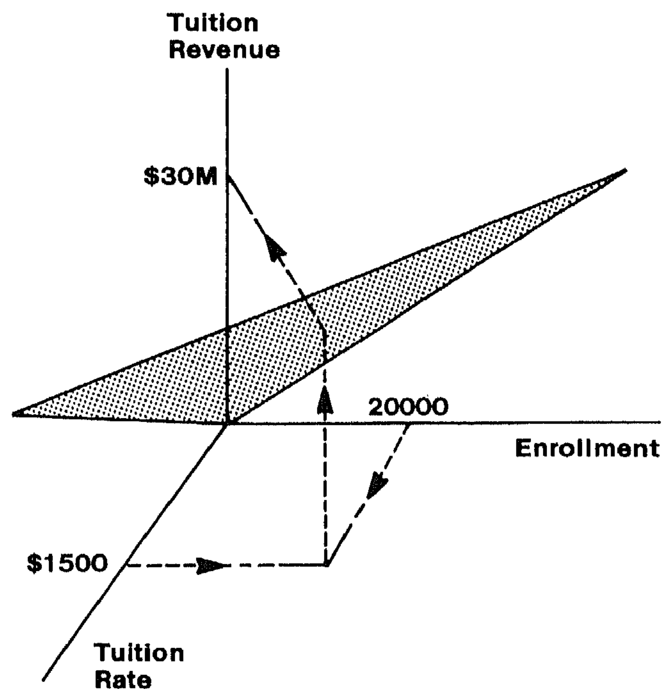

Figure 1. A three-dimensional graph.

This method, already cumbersome at three variables, becomes impossible if we must consider tuition revenue to be a function of *three* independent variables, at which time a four-dimensional graph would be required.

The four-dimensional computational diagram. All the preceding difficulties disappear once we abandon the traditional format of graphing variables along orthogonal axii. Figure 2 is such a graph, one which we choose to call a "computational diagram." This diagram relates four variables: workload in contact hours/FTE (on the left-hand vertical axis), workload in student credit hours/FTE (on the curved axis), the number of students admitted each year (on the horizontal axis), and the number of faculty FTE (on the flared diagonal axis).

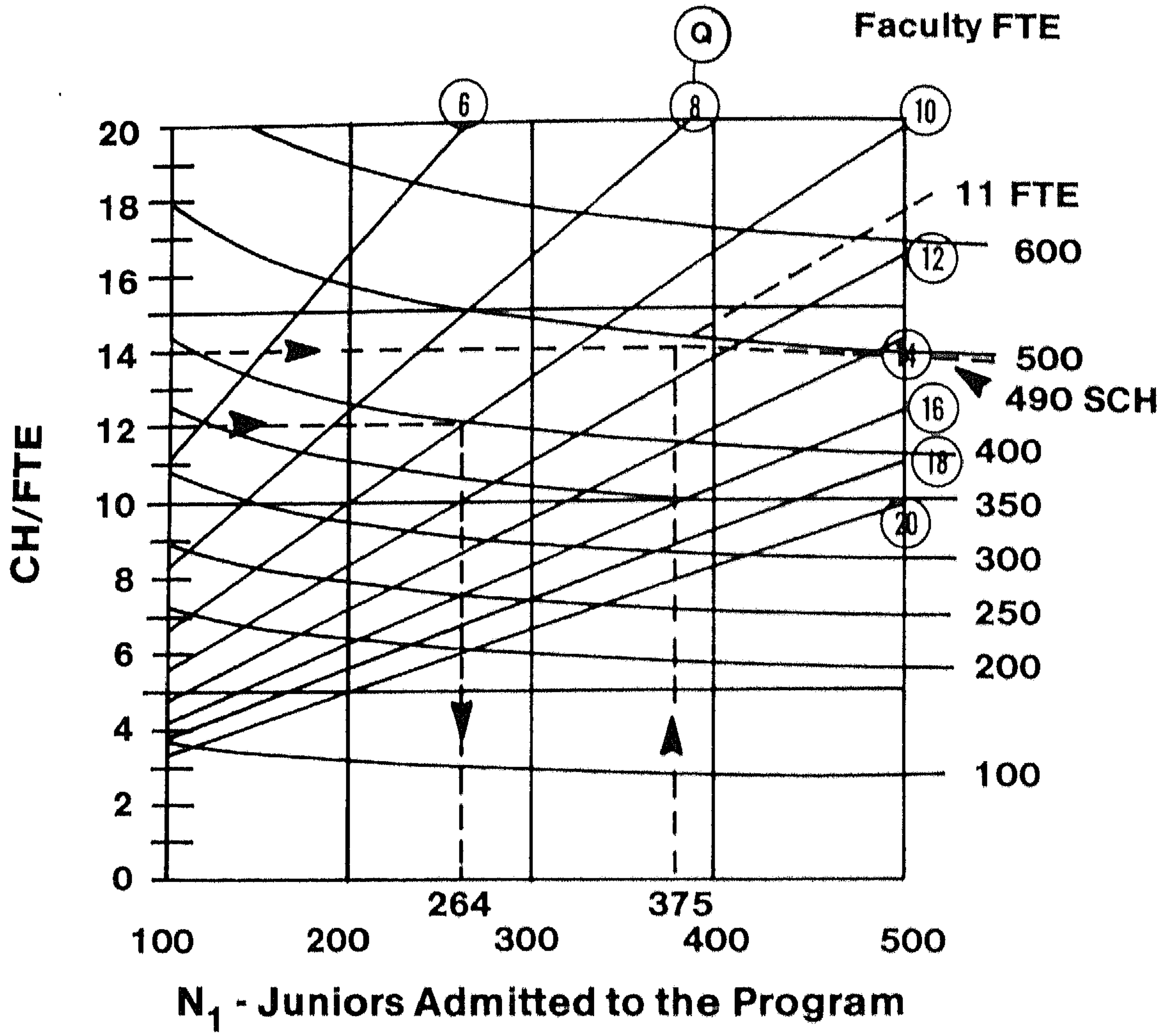

Figure 2. Four-variable computational diagram for analyzing a technical/professional program.

Figure 2 may be used in a number of ways. For instance, if we wish to maintain a 12-contact-hour/FTE/term workload and have 10 faculty FTE, we draw a horizontal line at the 12/CH/FTE/term level until it intersects the 10 FTE diagonal. From this intersection, we drop a vertical line to the horizontal-scale intersection and find that about 264 juniors may be admitted each year. We also learn that the workload level under these conditions will be approximately 400 SCH/FTE (because the intersection lies near the 400 SCH/FTE scale curve).

This diagram may also be used to answer the question, How many faculty do we need if we want to admit 375 new juniors per year and maintain a 14-contact-hour/FTE workload? When we draw a horizontal line at 14 CH/FTE/term and a vertical line at 375 new juniors, the intersection lies slightly more than half way between the 10- and 12-FTE, diagonal-scale lines, indicating that about 11 FTE will be needed to maintain the required student population and workload. We also find that this situation will require a faculty workload of about 490 SCH/FTE.

Figure 2 was developed from a model based on a detailed analysis of a department's teaching model, laboratory limitations, course structure, and staff configuration, the details of which are given in the next section.

A workload model. As undergraduate program preferences change, it is often necessary to consider enrollment limitations for programs and departments within the university. This is especially true in times of economic troubles and budget reductions. In order to accomplish a realistic yet understandable analysis of departmental workloads, student numbers, and faculty strength, a mathematical model relating these factors, and others, was developed by the authors.

The basic unit of this departmental model is the course. The workload contribution and student enrollment are modeled for each course offered by the department over a selected time interval, in this case the three-term academic year. Several examples will illustrate this:

Example 1. EPD 200 is a four-credit course offered each term. The lecture component of this course meets for three one-hour sessions a week in sections of X_{200}; the laboratory component meets once a week for three hours in sections of X_{200L}. Usually, half of the juniors and seniors in the academic program are enrolled in

EPD 200 during any given year. CH and SCH are given by

$$CH_{200} = \left(\frac{N_1 + N_2}{2}\right) \cdot \left(\frac{3}{X_{200}} + \frac{3}{X_{200L}}\right)$$

$$SCH_{200} = \left(4 \frac{N_1 + N_2}{2}\right)$$

where N_2 = number of seniors in the department's program and may be related to number of juniors by

$$N_2 = (1 - \alpha)\, N_1(t-1)$$

where $N_1(t-1)$ is the number of juniors admitted the previous year and α is a drop-out fraction. (X_{200} may equal 60, and X_{200L} may equal 20 students.)

In Example 1, we have examined a course which is considered to be in the "core" of the academic program and is open only to students admitted to the program.

Example 2. EPD 100 is a "service course" open to all university students. It is a three-credit course taught in three one-hour lectures each week, one large lecture section each term. The CH and SCH may be given by

$$CH_{100} = 9$$

$$SCH_{100} = 541 + 9.47N_1$$

This SCH equation was developed using historical data of *service SCH* vs. *number of juniors* in the program. A least-squares regression was used to fit a straight line to the (SCH, N_1) points, thereby relating the service SCH to historical student demand for the program.

All courses in this particular department's program—core, service, elective, and graduate—were modeled using the methods and combinations of methods outlined in the previous examples. Total contact hours and total SCH were then found by summing over all courses:

$$CH_{Tot} = CH_{100} + CH_{200} + \ldots$$

$$SCH_{Tot} = SCH_{100} + SCH_{200} + \ldots$$

Now that the two workload variables may be viewed as functions of admitted juniors, section sizes, and other teaching-model parameters, the workloads may be related to the FTE count of the faculty (Q). (All course and staffing information is normalized for a three-term academic year.) These are the relationships shown in Figure 2.

Constructing a four-dimensional computational diagram. The following is a step-by-step description of the procedure used to generate the diagram shown in Figure 2. This process may be generalized for any similar four-dimensional relationship among variables.

1. Determine all model assumptions exactly. Set all model parameters (section sizes, for example) at constant values except for the four parameters of interest: CH/FTE, SCH/FTE, N_1, and Q (N_1 = number of juniors admitted, Q = faculty FTE).
2. Determine which two variables are to be used for the orthogonal axii. For our example, CH/FTE was placed on the vertical axis with a range of 0 to 20, and N_1 was placed on the horizontal axis with a range of 100 to 500 (N_1 min to N_1 max).
3. Determine the range of values to be shown on the Q scale; Qmin to Qmax (6 to 20 on Figure 2).
4. Set Q = Qmin and incrementing N_1 between N_1min and N_1max; calculate values for CH/FTE from the model's equations. This produces a series of (CH/FTE,N_1) points. Connect these points to produce the Qmin scale line.
5. Increment Q by a convenient value, Qinc (Qinc = 2 in Figure 2) and repeat Step 4. Continue this process through Q = Qmax. This produces the Q scale.
6. Determine the minimum and maximum values to be used for the SCH/FTE scale (SCH/FTEmin, SCH/FTEmax) and an appropriate increment between scale lines, SCH/FTEinc. (SCH/FTEinc = 50 and 100 in Figure 2.)
7. Set SCH/FTE = SCH/FTEmin and Q = Qmin. Rearranging the model's equations, compute N_1 (SCH/FTEmin, Qmin); this is the point (N_1, Qmin), the intersection of the N_1 scale line with the Qmin scale line. Increment Q and recompute the point (N_1, Qmin + Qinc). Continue this process until N_1 = N_1max. Connect the N_1,Q) points found in this step to obtain the SCH/FTEmin scale line.
8. Increment SCH/FTE (SCH/FTEmin + SCH/FTEinc), repeat the process in Step 7, and draw the second SCH/FTE scale line. Repeat this process until the SCH/FTE scale is complete.

Figures 3 and 4 are additional computational diagrams based on a different modeling procedure and different teaching models. These diagrams were used successfully in a planning negotiation between the provost and the involved dean and chairpersons; the procedures, models, and assumptions were agreed to by all parties involved in negotiating a solution. Figure 2 is a model of what could be referred to as a technical/professional curriculum; Figure 3 was developed for a classical engineering program with almost no service component. Figure 4 is for a quantitative, engineering-related discipline with no engineering laboratories but with a very large service component. These diagrams may be used exactly as was Figure 2. Clearly, the underlying model may have to be quite complex in order to represent all the relevant variables; however, the resulting computational diagram is generally very simple in structure, and workload measures and enrollments may be quickly traded off against one another to reach a solution that is acceptable to all participants in a planning session.

The value of such diagrams becomes apparent in those situations where we must make a series of repetitive calculations, when an equation must be solved for different variables, or a complex computer simulation must be rerun. Such a situation may be exemplified by a planning session where the participants have only a vague idea of what should be done and prefer to arrive at an understanding by asking a series of "what if" questions. In the previous examples, planners could readily calculate their own answers to questions like these: If our eight faculty were to teach sixteen hours a week, how many juniors could we accept into the program? or, If we want to hold the teaching load to

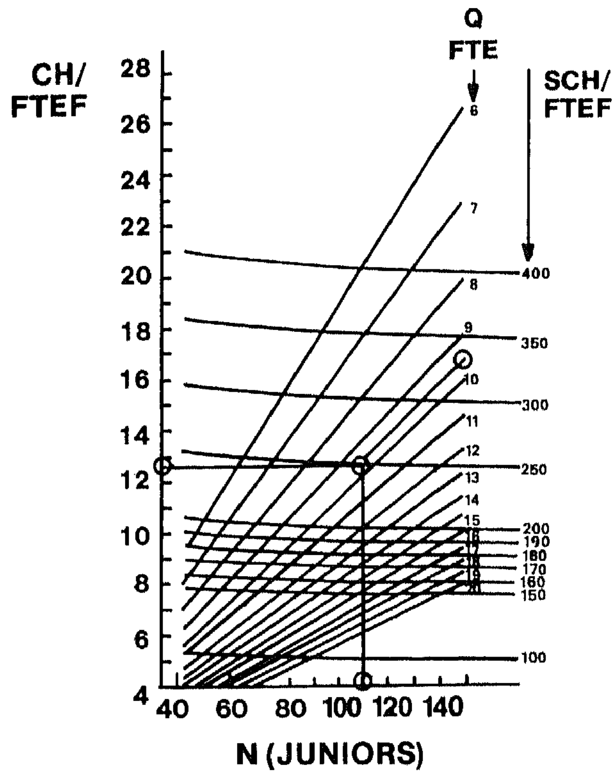

Figure 3. Computational diagram for an engineering program with no service component.

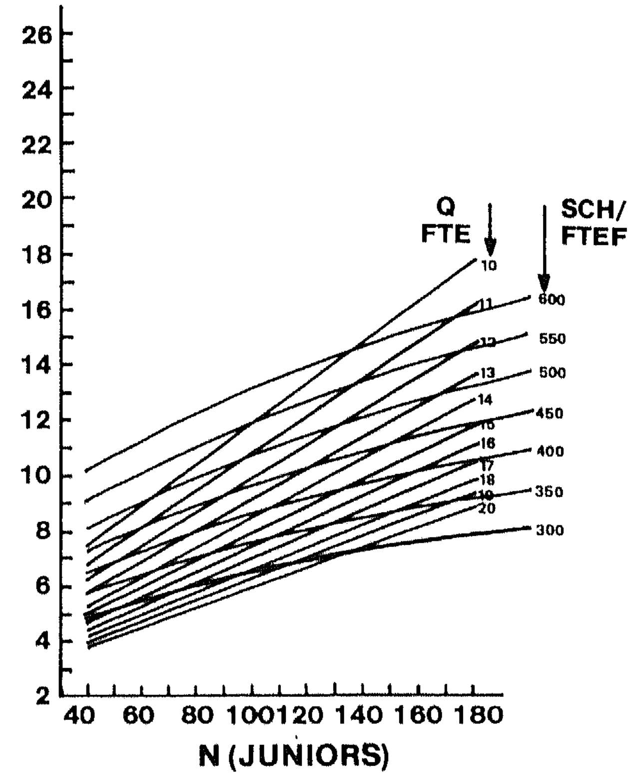

Figure 4. Computational diagram for an engineering discipline with no laboratories.

twelve hours a week, how many faculty will we need to handle our 120 juniors? The fact that a computational diagram can easily be used by *all* the participants to obtain nearly *immediate* answers to their questions is clearly preferable to the more common situation where technical questions are referred to a specialist who responds days later. In this milieu, the computational graph serves the same purpose as a simple computer model that can be used interactively by all participants on their own terminals.

It seems clear that diagrams of this nature are not limited by the complexity of the calculations as much as by the number of variables. Four or five variables can be accommodated within a readable format. Beyond that, the researcher must depend upon his or her ingenuity to make it understandable. Typically, however, administrators and planners establish reference points by examining the effects of a few commonly used and accepted variables, a situation easily handled by our computational diagrams.

Nomographs

In the foregoing, we have shown that relatively simple calculations such as frequently occur in higher education administration can easily be performed using specially constructed graphs. We now illustrate yet another method to perform calculations, the nomograph, which has characteristics that make it more useful, in certain instances, than a computational diagram.

At first glance, the nomograph may appear to be indistinguishable from the computational diagrams already discussed, but the underlying principles are quite different. In essence, a nomograph involves a set of numerical scales calibrated along straight lines which are, in most cases, parallel. Unknown values are found by using a straightedge to line up several given values and reading the unknown value located at the intersection of the straightedge and the scale of the unknown variable. Thus, in the nomograph in Figure 5, a given value of A=5 and B=10 would result in C taking on the value 400, where the dotted line represents a straightedge used to align points that are all related by a function linking A, B, and C.

Nearly any calculation that can be represented by a computational diagram can also be represented by a nomograph, or conversely. In general, nomographs work best when the number of variables is less than six and the mathematical relationships are relatively simple, (i.e., involving arithmetic operations). Since the nomograph uses only the intersection of a line with other calibrated lines, the graph is free of the confusing clutter of curves and the horizontal and vertical grid lines associated with graph paper.

In addition, the nomograph technique enables one to obtain accurate results easily, without the need to make rough interpolations between curves, as is often the case with the use of computational diagrams. A final advantage of the nomograph is that a step-by-step construction procedure can be formulated. The computational diagram has greater versatility (i.e., it can handle more complex relationships involving many variables), but it is more difficult both to use and to construct.

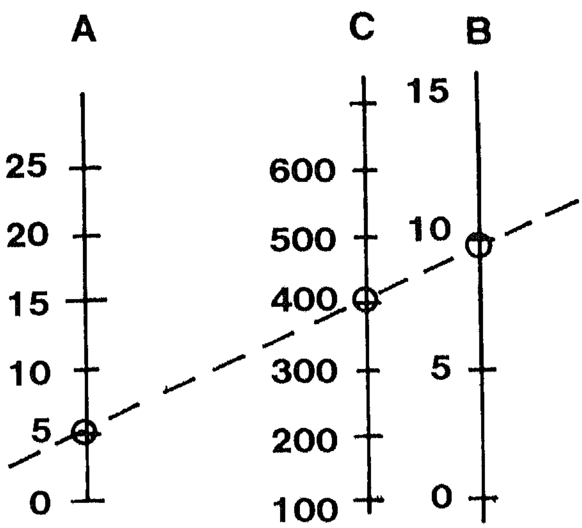

Figure 5. Using a simple nomograph.

As an illustration of the advantages of the nomograph, consider Figures 6a and 6b, which represent the instructional model of a chemical engineering department which has 110 entering juniors and a faculty of 9.5 FTE. In Figure 6a, the darker lines indicate that the ensuing workloads would be 250 SCH/FTE and 12.5 Contact Hrs/FTE. The same solutions can be gained from the nomograph in Figure 6b. The 9.5 FTE on the far left axis and the 110 juniors on the right axis are connected by a straight line that cuts the transversal axii at 12.5 and 250. A comparison of the two graphs will disclose some of the pros and cons.

Note how quickly this procedure, using either graph, can be carried out with a few practice trials; in fact, one could perform these mechanical movements more quickly than he/she could keypunch the input values of N and Q into a preprogrammed minicomputer or hand calculator. Certainly, using the nomograph is far faster than solving the original equations with a calculator. Finally, it should be clear that we do not always have to use N and Q to find SCH/FTE and CH/FTE. Given any pair of the four variables (except SCH/FTE and CH/FTE), we can find the remaining two by constructing the tie line using the two given values.

Constructing nomographs. Nomographs were invented in 1899 by a French mathematician, Maurice D'Ocagne, and quickly became a standard technique in the scientific and engineering fields. Most of the more useful nomographs were soon constructed, and the practice of nomography dropped from usage as quickly as it had begun, disappearing from the engineering curriculum by 1930. As a result, the more useful textbooks on this technique are well over fifty years old.

Most textbooks on nomography proceed by developing nomographs for certain fundamental equations. Once these standard forms are mastered, the technique is quickly generalized to more complex equations by viewing these equations simply as combinations of the fundamental forms. A quick summary of this approach is useful for gaining a general understanding of nomograph construction.

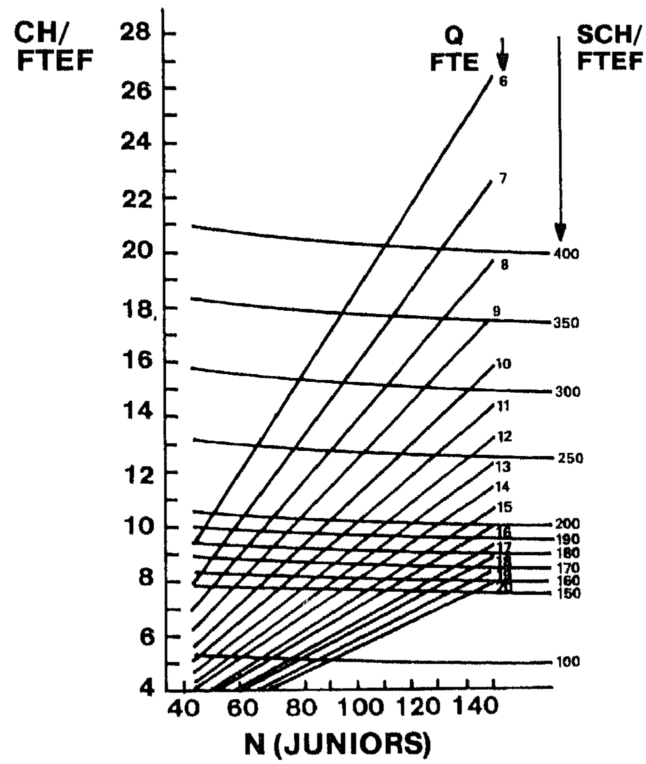

Figure 6a. Representation of the instructional model of a chemical engineering department, using a computational diagram.

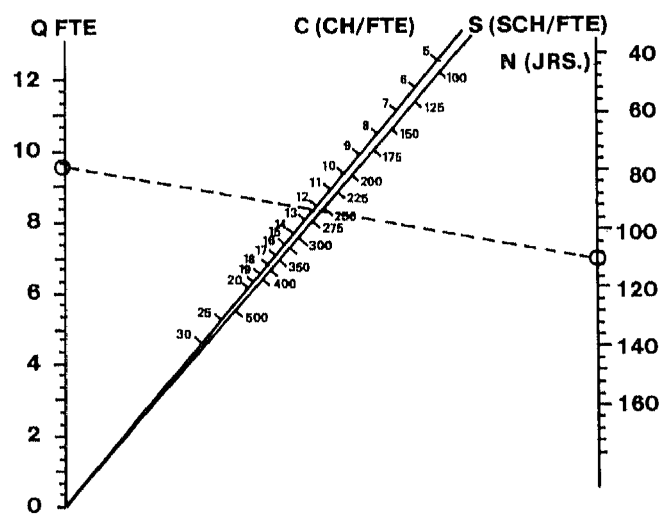

Figure 6b. Representation of the instructional model of a chemical engineering department, using a nomograph.

The first standard form is the equation $f_1(p) + f_2(q) = f_3(r)$ where f_1, f_2, and f_3 are *functions* of p, q, and r respectively. An example would be this:

$$\underbrace{(p-1)^2}_{f_1(p)} + \underbrace{q^2}_{f_2(q)} = \underbrace{(3r+2)}_{f_3(r)}$$

The characteristic nomograph associated with this equation is shown in Figure 7. The problem of construction becomes one of properly calibrating the p, r, q scales and determining the distance between them.

The second standard form is $f_1(p) \cdot f_2(q) = f_3(r)$ and has the characteristic form, as shown in Figure 8.

These two standard forms would suffice for a good many equations applicable to higher education. When used in combination, the possibilities are greatly extended, as will be demonstrated in the concluding example.

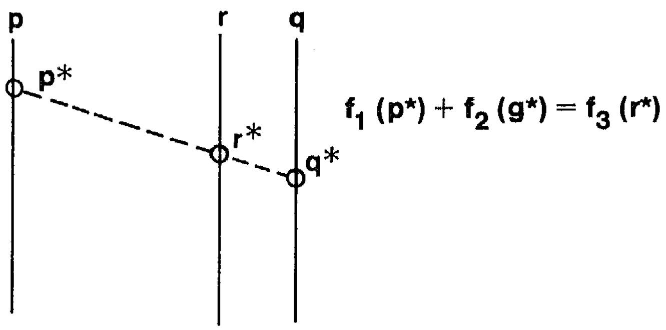

Figure 7. Nomograph for the basic equation, $f_1(p)+f_2(q) = f_3(r)$.

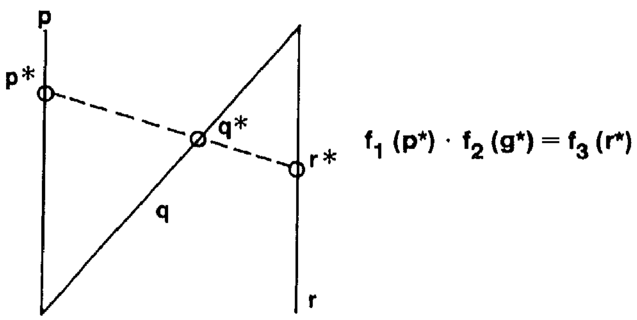

Figure 8. Nomograph for the basic form, $f_1(p) \cdot f_2(q) = f_3(r)$.

Example. Suppose we wish to construct a nomograph enabling us to find trade-offs between tuition increases and salary increases that will result in a balanced budget for a small state-supported college. Assume we are told that the current enrollment (E) is 10,000, the salary budget is \$60 million, the tuition averages \$2,000/year, additional revenues total \$20 million, and additional expenditures total to \$10 million. The current state appropriation is \$30 million.

Assume that enrollments, appropriations, tuition, and salary are all variables that can be changed.

Let: E = most current enrollment
A = % change in the state appropriation
T = % change in the tuition
S = % change in the salary budget

If the model is to reflect a balanced budget, we set:

revenue = expenditures
(tuition revenue + appropriation + other revenues) =
(salary budget + other expenditures)

$$E \times \$2{,}000(1+T) + \$30M(1+A) + \$20M = \$60M(1+S) + \$10M$$

Simplifying, we get:

$$\underset{f_1(E)}{\underset{\uparrow}{(2E)}} \cdot \underset{f_2(T)}{\underset{\uparrow}{(1+T)}} + \underset{f_3(A)}{\underset{\uparrow}{[10{,}000(3A-2)]}} = \underset{f_4(S)}{\underset{\uparrow}{60{,}000S}}$$

Thus, we want to construct a four-variable nomograph for the equation of type:

$$f_1(E) \cdot f_2(T) + f_3(A) = f_4(S) \qquad (1)$$

We handle this by writing it as two easy problems:

Let: $Q = f_1(E) \cdot f_2(T)$ (2)

Then, Equation 1 gives: $Q + f_3(A) = f_4(S)$ (3)

A nomograph can be constructed for each of these two equations, as in Figure 9. The final nomograph is formed by merging the two nomographs in Figure 9 by using a single Q axis (Figure 10). (Certainly the problem of plotting the precise position of the five scales and of calibrating them remains. These details are not difficult to solve, but they cannot be explained in a paper of this length. The point the authors wish to make with the example is that useful nomographs can be constructed using minimal theory.) The completed nomograph appears as Figure 11.

We expected the T scale to be a transversal line from the E to the Q scale, and yet it appears to be vertical. This is only because the E scale does not start at 0 but at

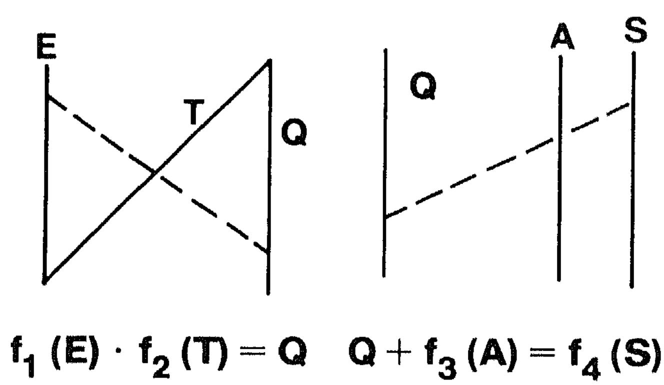

Figure 9. Nomograph format for equations (2) and (3).

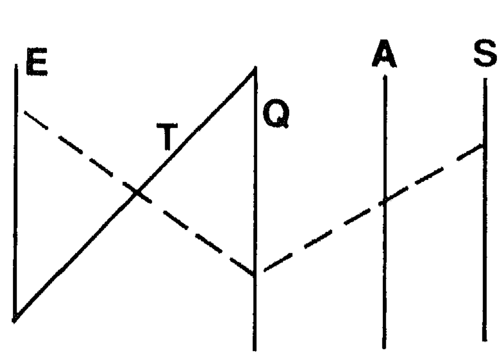

Figure 10. Nomograph format for $f_1(E) \cdot f_2(T) + f_3(A) = f_4(5)$.

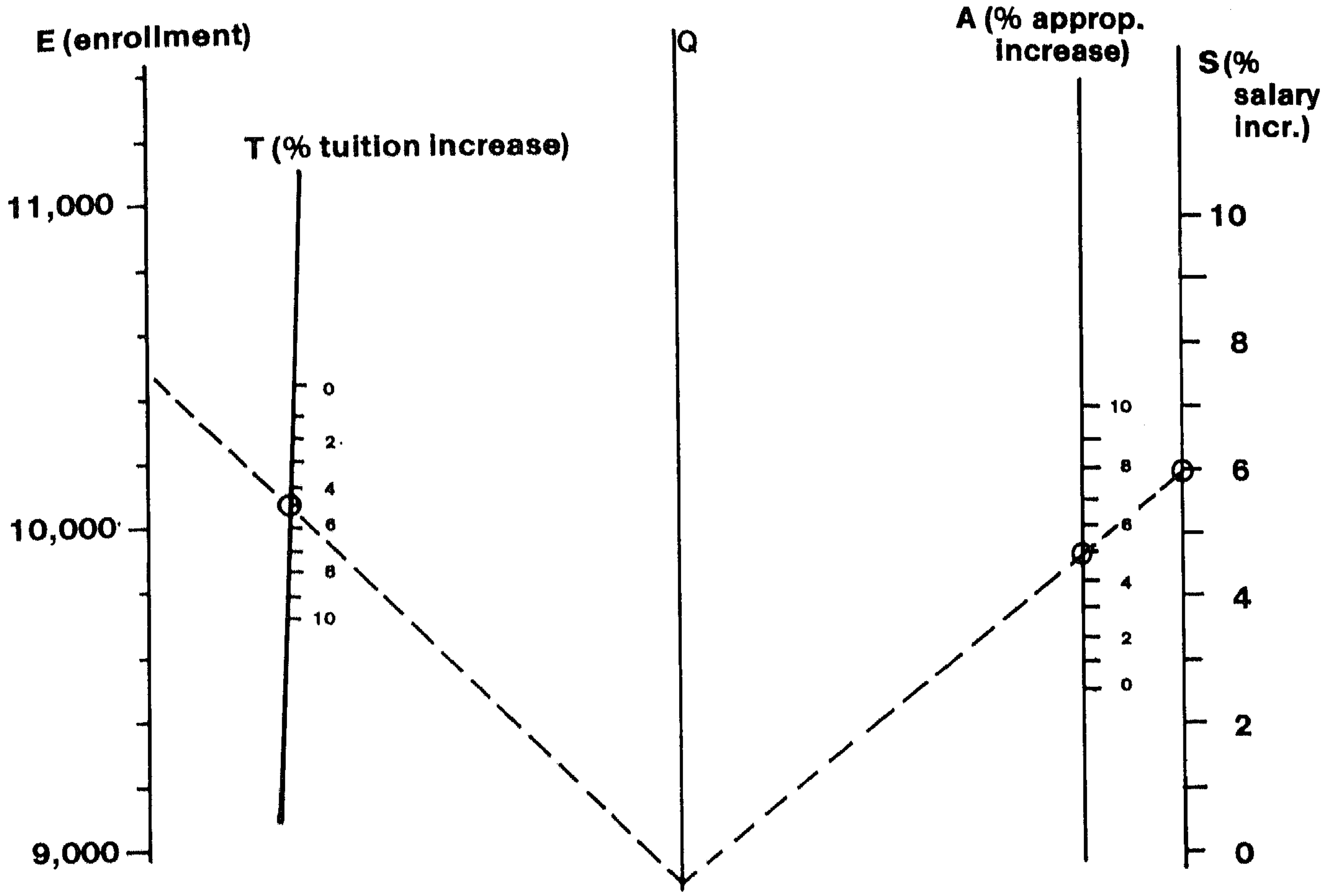

Figure 11. Nomograph of a state-supported college budget.

9,000; the slight incline of the T scale would cause it to cross the E scale at the 0 point, if extended.

A typical solution may proceed as follows: We are given that the expected appropriations will increase by 5% and that the enrollments will rise to 10,500. If we wish to increase salaries by 6%, what accompanying increase in tuition is required? The solution indicated by the dotted tie lines on Figure 6 is approximately 5%. (Numerical calculations produce a figure of 5.2%.) Repeated trials can be performed to give immediate answers to similar "what if" questions and to arrive at a consensus as to the proper course of action. Precise numerical calculations can then be made to refine the selected percentages.

Summary

The authors have attempted to show that sophisticated calculators and microcomputers have not yet relegated all older computational techniques and devices to the dust bin. There are still certain situations in which older methods hold an advantage in both speed and convenience. The slowest step in any calculation for which a calculator or computer is used is the human hand that keys in the data. As we have demonstrated, once a calculation involving several variables has been represented by a computational diagram or a nomograph, the user often can obtain solutions more quickly than is possible by computer. The first task is the identification of those special situations that warrant the use of diagrams and nomographs. In higher education administration there are at least two. One such instance occurs wherever a routine calculation must be performed repeatedly, using different input values. The other situation is more difficult to recognize; it will most likely develop within a planning milieu, where a relatively simple analytical model is needed to calculate responses to "what if" questions posed in the form of values assigned to input variables. A fast response is essential. In this situation, a computational diagram or nomograph reveals its full potential. These efficient devices not only equal the performance of a microcomputer; they are also more convenient to carry and, most importantly, they can be used by all participants in the planning session. More than one user has expressed delight with the simplicity and cleverness of these tools, an initial reaction that swiftly leads to a general acceptance of results. Experienced modelers will appreciate the psychological advantages that accrue when a planner can use a model, in the form of a nomograph or computational graph, to answer his or her own questions—a situation that is difficult to achieve if computer models or complex equations are used.

Once an alert administrator or researcher has discerned a situation conducive to the use of a nomograph or computational diagram, the task of its construction remains. The construction is not difficult and can readily be mastered with the aid of one of the suggested texts.

Suggested Reading

Nomography

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